

A Fibonacci atomic chain with side coupled quantum dots: crossover from a singular continuous to a continuous spectrum and related issues

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Interaction of bound states with a singular continuous spectrum is studied using a one dimensional Fibonacci quasicrystal as a prototype example. Single level quantum dots are attached from a side to a subset of atomic sites of the quasiperiodic chain. The proximity of the dots to the chain is modeled by introducing a tunnel hopping between a dot and the backbone. It is shown that, depending upon the proximity of the side coupled dot, the spectrum of an infinite quasiperiodic chain can display radical changes from its purely one dimensional characteristics. Absolutely continuous parts in the spectrum can be generated as well as isolated resonant eigenstates whose positions in the spectrum are sensitive to the proximity of the quantum dots. The cycles of the matrix map and the two terminal transport are discussed in details.

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I. INTRODUCTION

The physics of the condensed matter and materials science in the last couple of decades has been largely dominated by the mesoscopic and nano-scale systems [1]-[3]. The advancement of lithographic techniques together with the use of the instruments such as the scanning tunnel microscope (STM) has enabled experimentalists to examine the physical properties of tailor made geometries and test their potential as future nano-electronic devices. Such studies have been important not only from the point of view of possible applications, but also due to the fact that they play a crucial role in understanding the effect of quantum coherence in the electronic transport as the size of the system is reduced below the phase coherence length of the electrons [1].

A large section of the existing literature in this field deals with the effect of the quantum dots (QD), single, or an array of them, coupled from a side, on the spectral and transport properties of quantum nano-wires (QW) or quantum rings (QR) [4]-[16]. The studies include the investigation of the effect of inter-dot coupling in a side coupled double dot system [4], the tunability of the Fano-Kondo effect in a double QD unit [5], and a variety of phase coherent electronic transport studies [6]-[16]. The theoretical studies, in several cases, have been motivated by experiments on electronic transport in QW systems that were coupled to QD's from a side [8, 13].

One ubiquitous phenomenon that is manifestly evident in all such studies on quantum transport is the occurrence of Fano effect [17]-[21]. The Fano effect arises when a bound state 'interacts' with a contin-

uum, and is typically observed in the transmission spectrum of one dimensional QW systems when a single QD, or a cluster of them is attached to the QW from one side [14, 15, 20]. The transmission spectrum is marked with asymmetric lineshapes dictated by the formula [17],

$$\mathcal{F}(\epsilon) = \frac{(\epsilon + \eta)^2}{\epsilon^2 + 1} \quad (1)$$

where, $\epsilon = (E - E_R)/(\Gamma/2)$ with E_R being the 'resonance energy' and Γ the line width. η is the asymmetry parameter.

In spite of the considerable volume of work existing in this field, a practically unaddressed issue, to the best of our knowledge, is how seriously does the presence of a *bound state* caused by the attachment (from one side) of a QD or an assembly of them, influence a singular continuous spectrum. The localized state(s) can in principle, be seated anywhere in the spectrum, and a singular continuous spectrum having a multifractal distribution of gaps may be severely affected by these. This is the central motivation behind the present work.

We choose to work with a Fibonacci quasiperiodic chain that is a classic example of a *one dimensional* quasicrystal presenting a singular continuous spectrum [22]-[29]. Quasicrystals have been established as the third *ordered* phase of the solid state [22]. These are systems that are intermediate between a perfectly periodic system, and a completely random one. Over almost three decades, the physical properties of these strange systems have remained under active consideration, both from the standpoints of fundamental physics, and technological applications [23]-[29].

Spectral properties of a Fibonacci quasicrystal (FQC) are exotic. For example, its energy spectrum, in general, is singular continuous (a Cantor set) with measure zero. The spectrum exhibits a variety of scaling behavior [23]-[30]. The wave functions are neither periodic in the Bloch sense, nor are they exponentially localized as it happens in a completely random sequence of potentials [31].

Such peculiarities in the spectral properties have prompted researchers to propose and investigate realistic problems related to the localization of light in quasiperiodically ordered layered dielectrics [32, 33], flux pinning [34, 35], quasiperiodic optical lattices [36]-[38], plasmon excitation in aperiodically ordered dielectric layers [39], and many other cases. Experiments have also been performed to test the basic predictions of a purely one dimensional theory and to explore the novelties of these systems [40, 41]. The present day nanotechnology makes it possible to fabricate a lattice of QD's, and quantum wells with practically any desired geometry. Even stable and rigid (Carbon) atomic chains have been experimentally realized [42]. In several recent communications, aperiodically ordered metal nano-particle arrays [43]-[45], and aperiodic arrays of QD's [46] have been addressed. In every case, the quasiperiodic backbone plays a key role in controlling the physical properties of the system.

Such an environment has inspired us to undertake a detailed investigation in exploring the role of QD's side coupled to a Fibonacci quasiperiodic atomic chain. The dots are the *single level* QD's in the spirit of Kubala and König [47], and are attached to a subset of atomic sites in an infinite chain (Fig. 1a).

The results are quite extraordinary. Using a tight binding Hamiltonian for non-interacting, spinless electrons and a real space renormalization group (RSRG) scheme, we show that, when a single atomic site (equivalent to a single level QD) is attached to each of a subset of sites in the FQC, the energy spectrum can exhibit *absolutely continuous* subbands depending on the strength of the coupling of the side coupled atomic site to the Fibonacci backbone. The coupling represents the *proximity* of the adatom to the backbone, which can be controlled at will. There are other instances when the presence of adatoms can give rise to resonant (extended) eigenstates which were never present in the purely one dimensional model. As a result, a state that was *critical* [23] in the purely 1-d case, may turn out to be a resonant tunneling one, with a suitable proximity of the adatoms. The matrix maps [26], typical of a Fibonacci sequence, are also controlled by the

proximity of the adatoms. Such observations provide the first step towards answering the basic questions raised regarding the influence of bound states on a singular continuous spectrum. Consequently, the side coupled dots have profound effect on the two terminal electronic transport across a Fibonacci nanocluster. The system can be useful in designing possible tunnel devices with a quasiperiodic backbone.

In what follows, we describe our results. Section II contains the model and the principal methods of investigation. In section III we present the numerical results and discussion. Section IV elaborates the role of the adatom-backbone coupling in controlling the six cycles of the matrix map, and conclusions are drawn in section V.

II. THE MODEL AND THE METHOD

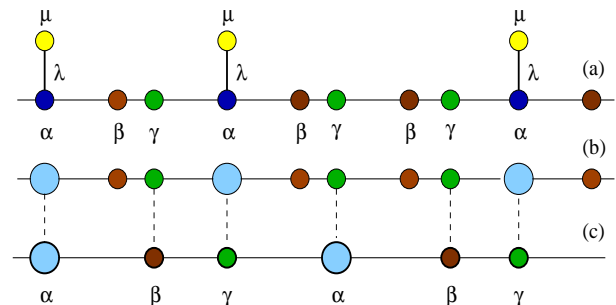


FIG. 1: (Color online). (a) A portion of an infinite Fibonacci chain with an adatom attached to every α -site. (b) The adatom is 'folded' into the backbone to generate an 'effective' α -site, and (c) the decimation renormalization scheme.

A. The Hamiltonian

To describe the system we use a tight-binding framework. In Wannier basis the Hamiltonian reads,

$$H = \sum_i \epsilon_i c_i^\dagger c_i + \sum_{\langle ij \rangle} t_{ij} [c_i^\dagger c_j + h.c.] \quad (2)$$

where, ϵ_i is the on-site energy of an electron at the site i and t_{ij} is the nearest-neighbor hopping strength. A binary Fibonacci chain comprising of two letters L and S is grown recursively following the growth rule [23] $L \rightarrow LS$, and $S \rightarrow L$, beginning with L . The Fibonacci family in successive generations appear as, $G_1 : L$, $G_2 : LS$, $G_3 : LSL$, and

so on. We consider the letters L and S to represent two kinds of *bonds*. The on-site potential ϵ_i takes on three values depending on its nearest neighbor configuration, viz, it is ϵ_α when flanked by two L -bonds on both sides, ϵ_β when it is between a L - S pair, and ϵ_γ for the S - L combination (see Fig. 1(a)). The nearest neighbor hopping is t_L and t_S when the electron hops across a L or a S bond respectively. The tunnel hopping, connecting a Fibonacci site and the adatom will be designated by $t_{ij} = \lambda$. The on-site potential of the side coupled dot is ϵ_μ .

B. The matrix and the trace maps

For the sake of completeness, let us remind ourselves the basics of the problem of determination of the eigenvalue spectrum and the eigenfunctions of a purely one dimensional FQC. It is efficiently handled by the transfer matrices M_ℓ corresponding to the ℓ th generation Fibonacci approximant [26]. The matrices of three consecutive generations are recursively coupled through the relation,

$$\mathbf{M}_\ell = \mathbf{M}_{\ell-2}\mathbf{M}_{\ell-1} \quad (3)$$

with $\mathbf{M}_1 = \mathbf{M}_\alpha$, $\mathbf{M}_2 = \mathbf{M}_\gamma\mathbf{M}_\beta$, and $\mathbf{M}_3 = \mathbf{M}_\alpha\mathbf{M}_\gamma\mathbf{M}_\beta$ [26]. The transfer matrices for the individual sites read,

$$\begin{aligned} \mathbf{M}_\alpha &= \begin{pmatrix} (E - \epsilon_\alpha)/t_L & -1 \\ 1 & 0 \end{pmatrix} \\ \mathbf{M}_\beta &= \begin{pmatrix} (E - \epsilon_\beta)/t_S & -t_L/t_S \\ 1 & 0 \end{pmatrix} \\ \mathbf{M}_\gamma &= \begin{pmatrix} (E - \epsilon_\gamma)/t_L & -t_S/t_L \\ 1 & 0 \end{pmatrix} \end{aligned} \quad (4)$$

The *allowed* eigenvalues are obtained from the condition $|x_\ell| \leq 2$ as $\ell \rightarrow \infty$, where, $x_\ell = \text{Tr}\mathbf{M}_\ell$, and is obtained recursively from the equation

$$x_{\ell+1} = x_\ell x_{\ell-1} - x_{\ell-2} \quad (5)$$

with appropriate initial values of x_ℓ depending on the model [26]. It is well known [23]-[29] that, as the generation index $\ell \rightarrow \infty$, every energy that one hits upon, corresponds to an escaping orbit of the trace map Eq. 5, and the spectrum turns into a Cantor set with a gap in the vicinity of every energy. Such a spectrum corresponds to eigenfunctions that are neither *localized* in an exponential way, nor are they *extended* in the Bloch sense. This fact is mathematically described by an *invariant* quantity given by,

$$I = \frac{1}{4}(x_\ell^2 + x_{\ell-1}^2 + x_{\ell-2}^2 - x_\ell x_{\ell-1} x_{\ell-2} - 4) \quad (6)$$

The above invariant becomes equal to zero for a perfectly periodic chain of atoms, while it is infinitely large when one calculates it for a randomly disordered 1-d lattice. The *zero* of the Fibonacci invariant thus corresponds to extended eigenfunctions [23], a fact that is crucial for our case.

C. The RSRG scheme

The self similarity inherent in the FQC structure makes the use of a real space renormalization group (RSRG) decimation scheme a natural choice to unravel the spectral features. The scheme is illustrated in Fig. 1. First, the side coupled QD's are 'folded' into the α -sites to create effective $\tilde{\alpha}$ -sites with potential $\epsilon_{\tilde{\alpha}} = \epsilon_\alpha + \lambda^2/(E - \epsilon_\mu)$. This process is shown in Fig. 1(b). The β -sites are then decimated to obtain a scaled version of the original chain, as shown in Fig. 1(c). The decimation method relies on the use of an infinite set of difference equations [48],

$$(E - \epsilon_j)\psi_j = t_{j,j+1}\psi_{j+1} + t_{j,j-1}\psi_{j-1} \quad (7)$$

with $\epsilon_j = \epsilon_\alpha, \epsilon_\beta$ or ϵ_γ as appropriate, and $t_{j,j\pm 1} = t_L$ or t_S . The renormalized values of the on-site potentials and the hopping integrals are given by,

$$\begin{aligned} \epsilon'_{\tilde{\alpha}} &= \epsilon_\gamma + \frac{t_L^2 + t_S^2}{E - \epsilon_\beta} \\ \epsilon'_\beta &= \epsilon_\gamma + \frac{t_S^2}{E - \epsilon_\beta} \\ \epsilon'_\gamma &= \epsilon_{\tilde{\alpha}} + \frac{t_L^2}{E - \epsilon_\beta} \\ t'_L &= \frac{t_L t_S}{E - \epsilon_\beta} \\ t'_S &= t_L \end{aligned} \quad (8)$$

The above set of recursion relations may be used to obtain the local density of states (LDOS) at any j -th site through the relation $\rho_j = (-1/\pi)\text{Im}G_{jj}$ where, G_{jj} is the local diagonal Green's function at the chosen site and, is obtained from the relation $G_{jj} = (E + i\eta - \epsilon_j^*)^{-1}$. $i\eta$ is the small imaginary part one needs to add to the energy E , and ϵ_j^* is the *fixed point* value of the relevant on-site potential, and is obtained from the set of Eq. 8, when t_L and t_S flow to zero under RSRG iterations [49].

D. The transmission coefficient

To obtain the two terminal transmission coefficient of an ℓ -th generation FQC, the sample is

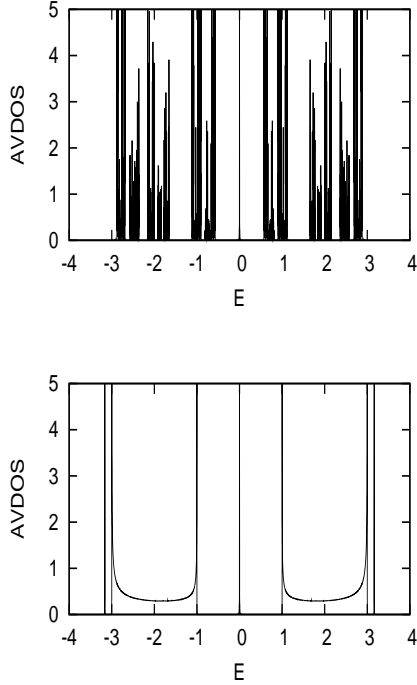


FIG. 2: Average density of states of an infinite Fibonacci quasicrystal in the transfer model, when, a single QD is attached to every α site. $\lambda = 1$ in the top panel and $\lambda = \sqrt{3}$ in the bottom panel. We have set $\epsilon_\alpha = \epsilon_\beta = \epsilon_\gamma = \epsilon_\mu = 0$, and $t_L = 1$ and $t_S = 2$.

clamped between two semi-infinite perfectly conducting leads [50]. The leads are modeled in the tight binding scheme by a uniform on-site potential ϵ_0 (set equal to zero everywhere in this calculation), and a constant nearest neighbor hopping integral t_0 (set equal to unity throughout). The transmission coefficient of the ℓ -th generation FQC is then obtained from the formula [50],

$$T = \frac{4 - E^2}{(Ez_\ell/2 - y_\ell)^2 + x_\ell^2(1 - E^2/4)} \quad (9)$$

where, $x_\ell = \mathbf{M}_\ell(\mathbf{1}, \mathbf{1}) + \mathbf{M}_\ell(\mathbf{2}, \mathbf{2})$, $y_\ell = \mathbf{M}_\ell(\mathbf{2}, \mathbf{1}) - \mathbf{M}_\ell(\mathbf{1}, \mathbf{2})$, and $z_\ell = \mathbf{M}_\ell(\mathbf{1}, \mathbf{1}) - \mathbf{M}_\ell(\mathbf{2}, \mathbf{2})$. The values of x_ℓ , y_ℓ and z_ℓ at any ℓ -th generation are obtained from their respective recursion relations, viz, $x_\ell = x_{\ell-1}x_{\ell-2} - x_{\ell-3}$, $y_\ell = x_{\ell-1}y_{\ell-2} + y_{\ell-3}$ and $z_\ell = x_{\ell-1}z_{\ell-2} + z_{\ell-3}$ with appropriate initial values [50].

III. RESULTS AND DISCUSSIONS

A. Absolutely continuous subbands

Let us refer to Fig. 1 (a). A single adatom marked μ and with on-site potential ϵ_μ is attached to every α -site of the chain. The FQC-adatom coupling is λ . The effect of the adatom is easily taken care of by defining a *renormalized* potential at the α -site (Fig. 1b). That is, the initial value of the potential at the α -site is not just ϵ_α , but, $\tilde{\epsilon}_\alpha = \epsilon_\alpha + \lambda^2/(E - \epsilon_\mu)$. With this modification, we work out the invariant (given by Eq. 6) of the trace map in Eq. 5 for a purely transfer model [26] with $\epsilon_\alpha = \epsilon_\beta = \epsilon_\gamma$, $t_L = \tau$, and $t_S = R\tau$. If we choose $\epsilon_\mu = \epsilon_\alpha$, the invariant remains independent of energy, and reads,

$$I = \frac{[\lambda^2 - (R^2 - 1)\tau^2]^2}{4R^2\tau^4} \quad (10)$$

This immediately leads to a situation where one can have a *zero* of the invariant independent of the electron energy E . The zero of the invariant in case of a FQC should correspond to extended eigenstates [23]. Setting $I = 0$ we obtain $\lambda = \pm\tau\sqrt{R^2 - 1}$. This gives us a measure of the *proximity* of the adatom for which one should get extended eigenstates in a FQC irrespective of energy.

Do these extended eigenstates form a *band*? To answer this question, we work out the commutator $[\mathbf{M}_{\gamma\beta}, \mathbf{M}_{\tilde{\alpha}}]$, where, $\mathbf{M}_{\gamma\beta} = \mathbf{M}_\gamma \mathbf{M}_\beta$, and $\mathbf{M}_{\tilde{\alpha}}$ are the transfer matrices for the $\gamma\beta$ cluster and the ‘renormalized’ α (now called $\tilde{\alpha}$) atoms respectively. The result is,

$$[\mathbf{M}_{\gamma\beta}, \mathbf{M}_{\tilde{\alpha}}] = \mathbf{0} \quad (11)$$

for $\lambda = \pm\tau\sqrt{R^2 - 1}$, independent of the electron energy E . It is to be appreciated that, this value of λ is the same as obtained by forcing the invariant to vanish. The vanishing of the commutator implies that, for the above choice of the tunnel hopping λ , the $\gamma\beta$ cluster and the $\tilde{\alpha}$ atoms can even be arranged in a periodically alternating manner, representing an ordered binary alloy. The energy band in this case consists of continuous distribution of eigenvalues with a gap separating the continuous sub-clusters. As a result of the commutation Eq.11, the electron will ‘feel’ no essential difference of the FQC with an ordered binary alloy, and the energy spectra of the original FQC should therefore be identical to that of a periodic arrangement of the constituent clusters $\tilde{\alpha}$ and $\gamma\beta$ placed in an alternating fashion. *Continuous subbands of extended states* in the spectrum, when $\lambda = \pm\tau\sqrt{R^2 - 1}$, is therefore an obvious result for the stubbed FQC.

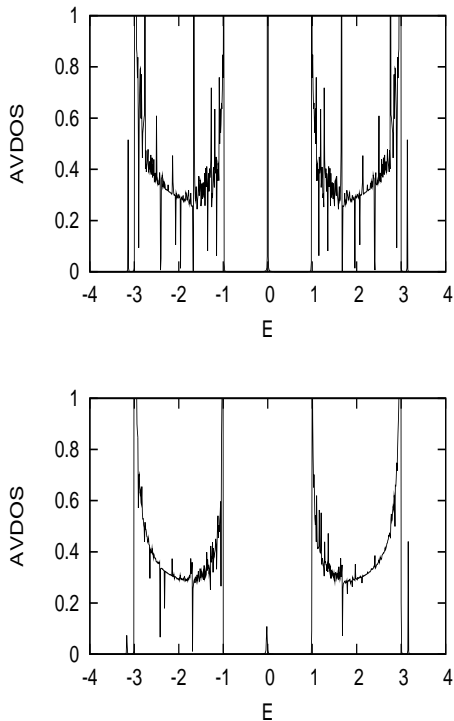


FIG. 3: Average density of states of a Fibonacci quasicrystal in the transfer model. A single QD is attached to every α site. We have chosen ($\lambda = 1.69736$, $\epsilon_\mu = 0$) (top panel), and ($\lambda = \sqrt{3}$, $\epsilon_\mu = -0.02$) (bottom panel) representing a 2% deviation from the exact values (with respect to Fig. 2 (bottom panel) of any one of the parameters λ and ϵ_μ respectively). Once again we have set $\epsilon_\alpha = \epsilon_\beta = \epsilon_\gamma = 0$, $t_L = 1$ and $t_S = 2$.

To confirm this we present in Fig. 2 the average density of states (AVDOS) of a FQC with adatoms attached to every α -site. We have set $\epsilon_\alpha = \epsilon_\beta = \epsilon_\gamma = 0$, $t_L = 1$ and $t_S = 2$ along the Fibonacci backbone. The site potential of the adatom is taken as $\epsilon_\mu = 0$, and the value of the FQC-adatom tunnel hopping λ has been set equal to 1 and $\sqrt{3}$ respectively in the two figures. For $\lambda = 1$ there appears a notable change in the spectrum compared to the usual three-subband spectrum of a standard FQC [26]. The sub-bands get shifted and the eigenvalues cluster in a different shape. There is a central peak at $E = 0$ that corresponds to a sharply localized eigenstate, and is a consequence of the attachment of the adatoms. But otherwise, the spectrum retains the typical fragmented, self similar character

of a 1-d FQC.

The remarkable part of the figure is the case with $\lambda = \sqrt{3}$ which corresponds to $I = 0$. The spectrum is *absolutely continuous* within two sub-bands. The central localized state remains pinned at $E = 0$ however. Two additional localized states show up immediately beyond the sub-bands on either side. The overall spectral character has been cross checked by explicitly working out the trace map. The completely gapless character of the spectrum in the range $-3 < E < -1$ and $1 < E < 3$ suggests that one should have all the eigenstates extended in these energy regimes when $\lambda = \sqrt{3}$.

The recursion relations Eq. 8 reconfirm the extendedness of an eigenstate corresponding to an eigenvalue falling within the continuum in the spectrum. The hopping integrals t_L and t_S remain non-zero under successive RSRG iteration for an indefinite number of loops if the chosen energy is picked up from within the continuum. This implies that, at any length scale there is a non-zero connectivity between the neighboring sites at that scale, and hence the corresponding state is of extended character [49].

Before we end this subsection, it's important to appreciate that, the origin of the continuous bands of extended states is the commutation of the matrices $\mathbf{M}_{\gamma\beta}$, and \mathbf{M}_α . Therefore, any quasiperiodic, or even disordered geometric arrangement of α (coupled to a single adatom), and the $\beta\gamma$ pair, under suitable conditions such as the above will lead to an identical energy spectrum and hence, extended eigenstates. We have tested this with other kinds of aperiodic chains which under RSRG give rise to various kinds of recursion relations between the Hamiltonian parameters. In every case, the bands with $\lambda = \sqrt{3}$ is the same. In this respect, such microscopically different aperiodic geometries can be brought under one *universality class*, using a suitable side coupled array of QD's. The transport behavior of course, is sensitive to the recursion relations, and its structural details will be different.

B. Stability of the continuous spectrum

A major concern from the standpoint of an experimentalist will be the stability of the continuum against a possible deviation of λ from its exact numerical value as obtained from Eq. 10, or a variance in the value of ϵ_μ . This is related to any error in fixing up the exact proximity of the adatom to the α -sites, or an error in controlling the potential of the attached QD by a gate voltage. To this end, we have extensively studied the AVDOS spectrum by vary-

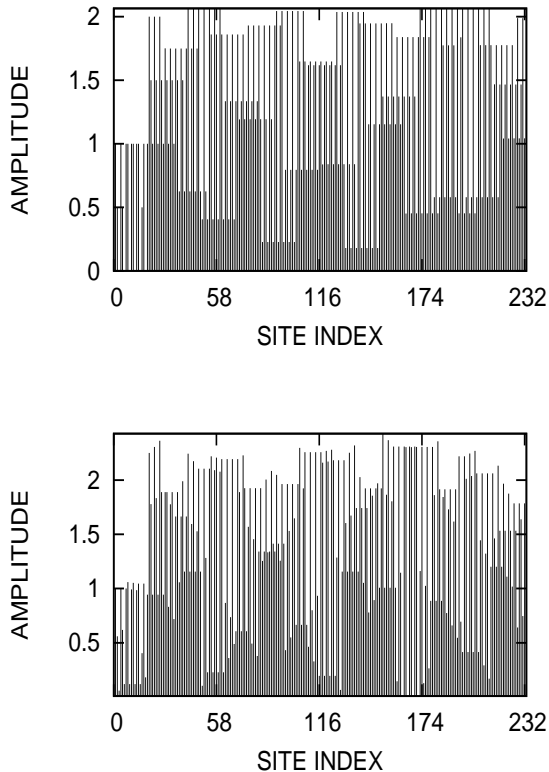


FIG. 4: $|\psi_n|$ plotted for 232 sites of a Fibonacci chain with adatom attached to every α -site. We have chosen $\lambda = \sqrt{3}$ (top) and $\lambda = 1.69736$ (bottom) showing that the extended nature of the wave function persists even with a 2% variation in λ . $\epsilon_\mu = 0$ in both the figures and $E = 2$, selected in the *continuous band* of the spectrum. Other parameters are the same as in Fig.2.

ing λ from its value of $\sqrt{3}$. For large deviations from $\lambda = \sqrt{3}$, the spectrum gets back to the familiar three sub-band Cantor set like structure typical to a FQC. However, for small deviations the continua persist, though with a ruggedness in the landscape, which becomes more and more dominant with increasing value of λ .

To illustrate, we have shown in Fig. 3 the AVDOS for $\lambda = 1.69736$, $\epsilon_\mu = 0$ (top), and for $\lambda = \sqrt{3}$, $\epsilon_\mu = -0.02$ (bottom) for the entire range of the spectrum. These values are deviations by 2% from the *critical* values, viz, $\lambda = \sqrt{3}$, or $\epsilon_\mu = 0$. The smooth AVDOS observed in Fig.2 (bottom) gets distorted by numerous oscillations in Fig. 3. But, the con-

tinuous distribution of eigenvalues still persists over smaller energy intervals. The patches of continua are also observed to survive even when the deviation is as large as 6% from either $\lambda = \sqrt{3}$, or $\epsilon_\mu = 0$, or even when both the parameters vary within a reasonable uncertainty in their values. This stability is also reflected in the amplitude distribution along the FQC backbone (Fig. 4). The energy is chosen to be $E = 2$, that is, from within a continuous subband. Oscillating but non-decaying profile of the amplitudes for arbitrarily large system size is observed. We have shown (top panel) the result for 232 sites, with $\epsilon_\alpha = \epsilon_\beta = \epsilon_\gamma = \epsilon_\mu = 0$, $t_L = 1$, $t_S = 2$, and for $\lambda = \sqrt{3}$. In the bottom panel we present the case where the FQC-adatom hopping λ deviates by 2% from its *critical* value of $\lambda = \sqrt{3}$. The extendedness prevails over extremely large system size, though we again show the results for just 232 sites here.

Thus, even within the natural experimental uncertainty, it should be possible to observe a crossover in the transport characteristics from the poorly conducting to a metallic one in a Fibonacci array with side coupled quantum dots, by controlling the proximity, or the potential of the dots with respect to the backbone.

C. The transmission coefficient

The observations made in the density of states spectrum are corroborated by the corresponding calculation of the transmission coefficient of finite but arbitrarily large systems using Eq. 9. In Fig. 5 we present the transmission spectra for $\lambda = 0$ (Fig. 5(a), no side atoms), $\lambda = 1$ (Fig. 5(b)) and $\lambda = \sqrt{3}$ (Fig. 5(c)) for the transfer model with $\epsilon_i = \epsilon_\mu = 0$, $t_L = 1$, and $t_S = 2$. The three sub-band clustering of the pure FQC in (a) evolves into a four sub-band form. $\lambda = \sqrt{3}$ clearly shrinks the entire spectrum into two continuous zones of high transmission coefficients. The sharply localized state at $E = 0$ of course, never contributes. This reflects what we have already discussed in the context of the density of states.

IV. SIX CYCLES OF THE MATRIX MAP AND THE ROLE OF λ

Extended eigenstates can also be traced which are related to a six cyclic behavior of the matrix map viz, $\mathbf{M}_{\ell+6} = \mathbf{M}_\ell$, $\ell \geq 1$ [23]- [26]. In a previous work [48] it has been shown that a six cycle of the matrix map given by Eq. 3 is caused by resonances occurring in

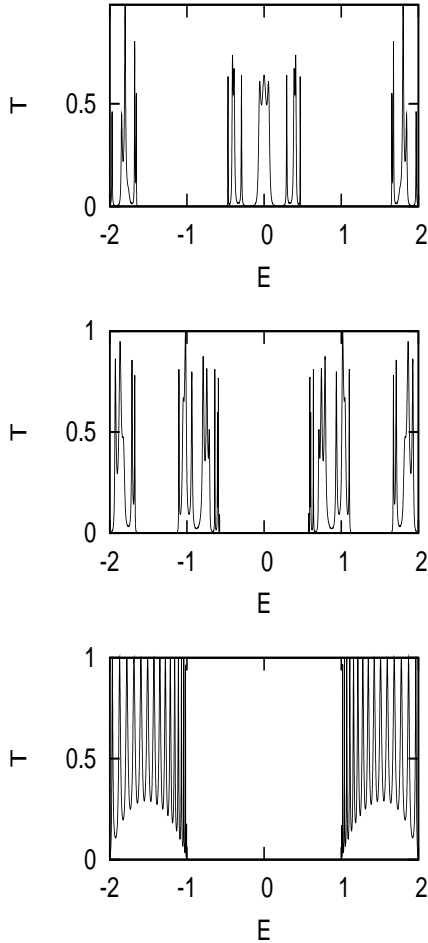


FIG. 5: Transmission spectrum of a 9th generation Fibonacci quasicrystal (55 bonds) with (a) no side attachment, (b) an adatom attached to every α -site in the bare length scale, and $\lambda = 1$, and (c) same as (b), but now with $\lambda = \sqrt{3}$.

the clusters $(\gamma\beta - \gamma\beta, \alpha - \alpha)$ in the bare length scale, or through the resonances taking place in pairs like $(\alpha\gamma\beta - \alpha\gamma\beta, \gamma\beta - \gamma\beta)$ in one step renormalized lattice. Bigger clusters are easily identified from higher order renormalized version of the FQC.

As already mentioned, in the six cyclic cases, $\mathbf{M}_\ell = \mathbf{M}_{\ell+6}$, for one or more special values of energy. The value of ℓ determines the length scale at which the six cycle of the map begins to show up, and depends on the construction of the resonating clusters [48]. The energy eigenvalue corresponding to the six cycle of the matrix map is extracted as a common root of the equations, $Tr\mathbf{M}_{\gamma\beta}(\mathbf{n}) = \mathbf{0}$, and

$Tr\mathbf{M}_\alpha(\mathbf{n}) = \mathbf{0}$. n denotes the stage of renormalization, and it provides the length scale at which the resonating clusters are identified.

In the FQC with attached QD's, the six cycle energies can be obtained almost at will by fixing the adatom at suitable places in the original lattice. For example, in the bare length scale $Tr\mathbf{M}_{\gamma\beta} = \mathbf{0}$ gives

$$E = \frac{1}{2} \left[\epsilon_\gamma + \epsilon_\beta \pm \sqrt{(\epsilon_\gamma - \epsilon_\beta)^2 + 4(t_S^2 + t_L^2)} \right] \quad (12)$$

while, the solution of $Tr\mathbf{M}_\alpha = \mathbf{0}$ gives,

$$E = \frac{1}{2} \left[(\epsilon_\alpha + \epsilon_\mu) \pm \sqrt{(\epsilon_\alpha - \epsilon_\mu)^2 + 4\lambda^2} \right] \quad (13)$$

One immediately finds that, for a given set of $(\epsilon_\alpha, \epsilon_\beta, \epsilon_\gamma, t_L, t_S)$, and for a certain value of ϵ_μ , one can tune the tunnel hopping λ so as to satisfy Eqs. 12 and 13 simultaneously. By choosing $\epsilon_\alpha = \epsilon_\beta = \epsilon_\gamma = \epsilon_\mu$, Eq. 12 provides $E = \epsilon_\alpha \pm \sqrt{t_L^2 + t_S^2}$ which is independent of λ . This energy value can be made equal to that obtained from the second equation, viz, Eq. 13 by choosing $\lambda = \pm \sqrt{t_S^2 + t_L^2}$. This gives an estimate of the *proximity* of an adatom to the α -sites in the original lattice that will be leading to six cycles of the matrix maps at special values of energies. We have extracted numerous such energy values from various scales of length. Each such energy corresponds to an extended eigenstate of the system in the sense that, the hopping integrals remain non-zero for an indefinite number of RSRG iterations.

V. CONCLUSIONS

In conclusion, we have examined the consequence of an interaction between bound states and a singular continuous energy spectrum by fixing isolated single level quantum dots as adatoms on specific lattice points of an infinite quasi-periodic Fibonacci array of atomic sites. The energy spectrum of system exhibits a remarkable transformation from a completely fragmented Cantor set nature with measure zero to one with continuous distribution of eigenvalues, similar to that of an ordered binary alloy, by judiciously choosing the QD-FQC tunnel hopping. One thus achieves an *almost insulating* to a metallic behavior of the system when the Fermi energy is located at suitable parts of the spectrum. The spectral crossover is found to be robust against a possible variation in the values of the tunnel hopping integral, or the potential of the adatoms. Other established properties of a FQC are also inspected and are found

to be sensitive to the values of the tunnel hopping which gives us an estimate of the proximity of the adatoms to the lattice. The two terminal transmission coefficient is evaluated using the standard recursive algorithm, and has been shown to corroborate our findings regarding the energy spectrum of a stubbed Fibonacci array of quantum dots. Detailed

analysis of the system at different scales of length can be made utilizing the real space renormalization group methods. This is under investigation, and the results will be reported in due course.

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